MÁS: Mathematics for **All** Students

Communication Is Critical

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Related professional history

- Austin ISD high school redesign and its foci on mathematics and on English language learners (2006)
- Classroom use of Schwartzman, S. (1994). *The words of mathematics*. Mathematical Ass'n of America.
- Foley, G. D., & Crocker, D. A. (2003). The words of mathematics. *The Centroid*, *29*(1), 8-9.
- Foley, G. D. (2004, March). *Inquiry-based mathematics*. Talk at the Legacy of R. L. Moore Conference, Austin, TX.
- Participation in AISD ELEVAR Cadre (2004-2005)
- Ideas from TSUS MELL, Dana Center, and the *Todos* Organization (2006-2007)

Overview

- Oral and written language skills are critical to success in mathematics.
- Language ties mathematical concepts and processes to everyday, social, and scientific applications.
- Mathematical reasoning, problem solving, and proof require sophisticated and technical communication skills.
- Both theory and research suggests a strong connection between mathematical success and student communication.

Principles & Standards

- *AMATYC standards addressed:* reasoning, connecting with other disciplines, **communicating,** developing mathematical power, linking multiple representations
- *AMATYC guiding principles addressed:* broadening, equity and access, inquiry, quantitative literacy, relevance, **research into practice,** technology
- *NCTM process standards addressed:* problem solving, reasoning and proof, **communication,** representation, connections

Key Words

- natural language
- conversational English
- academic English
- mathematics register
- *Tier I, Tier II, Tier III* words (Daro)
- **hyper-language** (Foley)
- ELL: English language learner
- LEP: limited English proficiency

Key References

- Elliott & Kenney (Eds.). (1996). *Communication in mathematics, K–12 & beyond: 1996 NCTM yearbook.*
- Rotman, B. (1988). Toward a semiotics of mathematics. *Semiotica*, *72*, 1-35.
- Schleppegrell, M. J. (in press). The linguistic challenges of mathematics teaching and learning. *Reading and Writing Quarterly*.
- Bay-Williams, J. M., & Herrara, S. (2007). Is "just good teaching" enough? In *The learning of mathematics: 69th NCTM yearbook.*
- Vygotsky, L. S. (1934/1987). *Thinking and speech.*

The mathematics register

The formal academic approach to mathematical speaking and writing is the **mathematics register.**

The mathematics register

- is complicated and multi-faceted,
- involves the many branches of mathematics, and
- uses the many symbol systems and representational modes of these branches.

Example of the mathematics register

Fun with matrix multiplication. A sequence of matrices based on repeated multiplication by the same square matrix is a **Markov chain**. Named for Russian mathematician Andrey (Andrei) Markov (1856–1922), Markov chains were first used in probability, but are now used widely in many fields. A Markov chain is a geometric sequence of matrices.

Example (cont'd.)

Using a graphing calculator for the computation, write out the first 7 terms of the Markov chain with first term [1 1] that is produced by repeatedly multiplying by the following transition matrix:

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

How is this sequence related to the Fibonacci sequence?

Sentence-by-sentence analysis (1)

“Fun with matrix multiplication.”

A casual, hopeful phrase but one that contains the mathematical term **matrix multiplication,** with which the reader may or may not be familiar. The reader may know the terms **matrix** and **multiplication** but may know little or nothing about **matrix multiplication.**

The reader may not realize that a working knowledge of matrix multiplication is assumed by the author and needed by the reader in order to continue successfully.

Sentence-by-sentence analysis (2)

"A sequence of matrices based on repeated multiplication by the same square matrix is a **Markov chain**."

An *informal* mathematical definition of the term **Markov chain,** indicated by the boldface type. A *formal definition* would be a full characterization of the concept, should be interpreted as an if-and-only-if statement, and could be used in mathematical proofs.

Mathematical writing is dense and replete with tacit assumptions.

Sentence-by-sentence analysis (3)

"Named for Russian mathematician Andrey (Andrei) Markov (1856–1922), Markov chains were first used in probability, but are now used widely in many fields."

An historical and biographical statement in academic English, with elements from both the mathematics register and the history register.

What knowledge and which conventions are assumed of the reader?

Sentence-by-sentence analysis (4)

"A Markov chain is a geometric sequence of matrices."

The reader is expected to use this statement together with "repeated multiplication by the same square matrix" to piece together a sufficient understanding of the term **Markov chain** to solve the problem that is about to be stated.

Sentence (4) will be of little help if the reader does not know the meaning of **geometric sequence** or does not call it to mind.

Sentence-by-sentence analysis (5)

"Using a graphing calculator for the computation, write out the first 7 terms of the Markov chain with first term [1 1] that is produced by repeatedly multiplying by the following transition matrix:

"

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

A complicated set of instructions to be carried out that draws on sentences (2) and (4), together with knowledge of sequence terminology and the use of a graphing calculator.

Sentence-by-sentence analysis (6)

"How is this sequence related to the Fibonacci sequence?"

A question calling for a comparative analysis of the results of the work just performed by the reader with the **Fibonacci sequence.** This assumes the reader has a working knowledge of the Fibonacci sequence.

The question is open ended, and it may not be clear to the reader what is expected.

Tier I, Tier II, and Tier III words

- **Tier I:** words used in everyday life, such as “chain" or “trousers," but like “chain" may have specialized meanings in the academic disciplines
- **Tier II:** general academic words such as "system" or “hypothesis," which are often critical for understanding mathematical speaking and writing
- **Tier III:** words of the academic registers, which are used for specialized disciplines, such as “medieval," “pulmonary," or "Markov chain"

(based on G. White’s description of Daro’s work, 2007)

The mathematics register

"In doing mathematics, it is not enough to be able to work with the language alone; mathematics draws on multiple semiotic (meaning-creating) systems to construct knowledge: symbols, oral language, written language, and visual representations such as graphs and diagrams."

(Schleppegrell, in press, p. 2)

The mathematics register

"Any mathematical text is written in a mixture of words, phrases, and locutions drawn from some recognizable natural language together with mathematical marks, signs, symbols, diagrams, and figures that (we suppose) are being used in some systematic and previously agreed upon way."

(Rotman, 1988, pp. 6-7)

Mathematics is a **hyper-language.**

Because the mathematics register is so multi-faceted and involves so many linked symbol systems and representational modes, I believe it should be thought of as a *hyper-language*.

The linked representational modes of the mathematics register

- Oral and written natural language
- Algebraic notation
- Tables of data and other numerical representations
- Cartesian and other coordinate graphs
- Diagrams and geometric figures
- Physical models and simulations

The peculiar linguistic patterns of the mathematics register

- Technical vocabulary
- Syntax of mathematical definitions
- Tacit and implicit logical relationships
- Use of the verbs *to be* and *to have*
- Technical meanings of *and* and *or*
- Complicated and dense noun phrases
- Use of inclusive and exclusive imperatives
- Uses of informal and formal language

Seeing Mathematics through the Lens of Communication

Knowledge and language are inextricably linked. Thoughts are closely tied to words. Mathematical concepts are expressed in words and phrases, and their meanings develop in the students' minds over time. Definitions, theorems, and other mathematical relationships and properties are expressed in natural-language sentences, often supported by mathematical symbolism, graphs, and other representations.

Rotman and Vygotsky

Rotman (1988) analyzes the tight interplay between a mathematician's thinking and the mathematician's "scribbling" in a mixture of words, symbols, and diagrams, and makes a strong case for the role of this written mathematical language in mathematical thinking and sense-making.

Vygotsky (1934/1987), on the other hand, focuses attention on the classroom and on shared oral language to develop ideas, which are then internalized individually.

Both written language and oral language are critical to the development of mathematical thinking.

NCTM Communication Standard

Instructional programs . . . should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

(NCTM, 2000, p. 60)

Austin ISD Enrollment, Fall 2006

Group	Count	Percentage
African American	10,450	12.73%
Asian	2,576	3.14%
Hispanic	46,852	57.09%
Native American	195	0.24%
White	20,028	24.40%
Total	82,074	

Austin ISD Enrollment, Fall 2006

Group	Count	Percentage
LEP English learners	20,782	**25.32%**
Spanish speaker subset	19,342	41.3% of Hisp.
Recent immigrants	3,834	4.67%
Spanish speaker subset	3,249	
Total	82,074	

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Bay-Williams & Herrara (2007)

Making mathematics accessible to ELLs requires focusing on the NCTM process standards together with "intentional language instruction and support" (p. 46)

"Literacy-rich classrooms foster ELLs' opportunities for simultaneous development of academic language proficiency and [mathematics] knowledge, skills, and capacities" (p. 45).

Teachers should combine vocabulary development and classroom discourse, with an "eye on language *and* mathematics" (p. 48).

Not just ELLs

Many students whose first language is English have limited English language skills. A student may be—

- fluent in a regional or ethnic dialect, but may lack interpersonal communication skills in standard American English.
- fluent in basic interpersonal communication skills in standard American English, but may have weak reading or writing skills.
- fluent in all four linguistic domains—reading, writing, speaking, and listening—of standard conversational English, but be weak in academic English.

Review

- Oral and written language skills are critical to success in mathematics.
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