Digging for Delusions

Math Misunderstandings

- Capability
- Procedural
- Conceptual

Capability Misunderstandings

- Not all students can learn math
- Men can understand math better than women
- Some people are born with a math gene

(Other such myths can be found in math anxiety lessons and books).

- *License plate numbers*

Procedural Misunderstandings: The student's view

- Math is about moving numbers around on pieces of paper to please your teacher who will give you an A and then you can forget the whole experience
- Doing math better means using more steps
- Neatness is more important than correct answers

Procedural Misunderstandings: The student's view

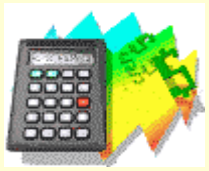
- Hardly anyone understands what they are doing or why

- If you are not good at memorizing rules find someone who is and have them do it for you

- Thinking is dangerous because it distracts you from the rules

Procedural Misunderstandings: The student's view

- No one knows who made the rules or why
- The use of tools invented after 1700 is illegal



- Since very few students can use the rules without errors, your grade will be based on how hard you try and whether the teacher likes you

Procedural Misunderstandings: The student's view

- About half the students will fail no matter how hard they try

- If you write a lot of different steps and answers you can hope one will be right

Procedural Misunderstandings: The student's view

- Cheating is an acceptable strategy because it may be the only way to pass and math is unfair anyway

- Math topics are separate and not related to each other

Arithmetic

Algebra

Geometry

Trigonometry

Procedural Misunderstandings: The teacher's view

- Students should sit separately and quietly listen to teacher's lecture

Procedural Misunderstandings: The teacher's view

Some methods of solving problems are better than others

Procedural Misunderstandings: The teacher's view

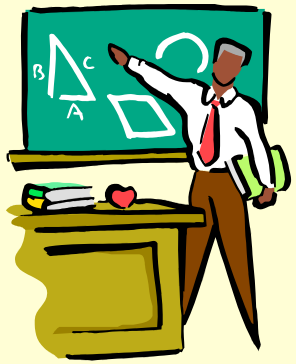
- Good students obediently and correctly memorize and use the math rules
- Good students never question what the teacher says

Procedural Misunderstandings: The teacher's view

- Students who attend regularly and on time, take notes, complete assignments neatly and on time, know the answers when called on, and can pass the tests are learning valuable math

Procedural Misunderstandings: The teacher's view

- Students can learn a lot of math by watching the teacher do problems on the board



Misconceptions

Why teach rules when we don't use them ourselves?

Describe the algorithm for multiplying 12 times 14

Will you use this algorithm for all numbers?

Will you use it for a septillion times a septillion?

Describe the algorithm for adding fractions

Add ¼ + ½

Did you use your algorithm for adding ½ + ¼ ?

Multiply 12 times 25 as quickly as you can in your head

How did you do it?

How would you find 15% of $240?

Is that the way you teach students to do it?

How would you do the following problem?

$(x+3)(x+5)$

Is FOIL the same as LOIF?

How would you do the following problem using the FOIL method?

$(x+y+3)(x+5)$

What is the first step to solve the following system of equations?

$$x + y = 5$$
$$2x + 3y = 13$$

#1

Why teach rules when we don't use them ourselves?

- Why not teach what we do so students will be as good as we are?
- We lead them astray
- “They are ruined by rules”
- They can never do it the way we do because they are afraid to leave the rules behind

#2

Maps are better than directions

Is anyone here from San Antonio?

When all you have is an algorithm, if something doesn't fit, you are lost. That's how our students feel. They are constantly nervous they will get off track and not be able to find their way back.

If all you have is a bunch of steps and procedures, if you make one mistake you are lost, and you don't even know when you have the correct answer because you don't know what it looks like.

We math teachers have maps but we don’t teach the students maps, we just give them directions.

We want to give students a map of the layout so they know lots of routes to the answer.

#3

Our students have misconceptions about math

Math is impossible to understand

- **An Unfair Test Question:**

What is 2/3 of 12?

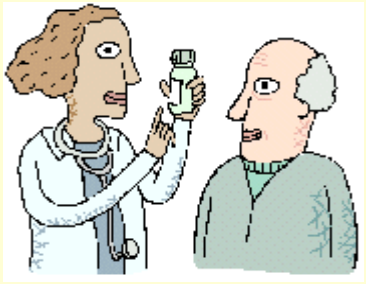
Math is about rules and procedures

- Juan and pi

Math is about rules and procedures

- **Nurse adding fractions:**

  ½ + ½ = ¼



Math is about rules and procedures

- “The first mark on a ruler is always 1/16”

Math is about rules and procedures

- Lecture notes: “Division is a shortcut for subtraction”

Math is about rules and procedures

ASAP Test Question:

The article from the Rainforest Action Network says that estimates of how much rainforest is disappearing vary, but you would like to examine at least one estimate to get some idea of how fast the tropical rainforests are disappearing. One source gives the following estimate of the size of Earth's rainforests for various years.

In 1980, there were 1,884,100,000 hectares of rainforest in the world.

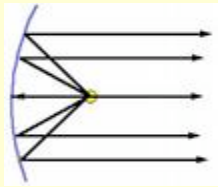
In 1990, there were 1,714,800,000 hectares.

(1 hectare = 2.47 acres or 0.00386 square miles)

Using this set of data, predict how many years it would take to totally destroy all of the remaining rainforests. Explain how you made your prediction. What assumptions have to be made in order for your prediction to be valid?

Real math is not about anything real

- Whispering Dishes



Real math is not about anything real

- Test tube 1/3 full

Real math is not about anything real

- **Walking rate**

If you start 5 meters from the wall and walk toward the wall at ½ meter per second, where will you be in 6 seconds?

Math is useless in real life

- District superintendent

Math is useless in real life

- 800 mg dilantin

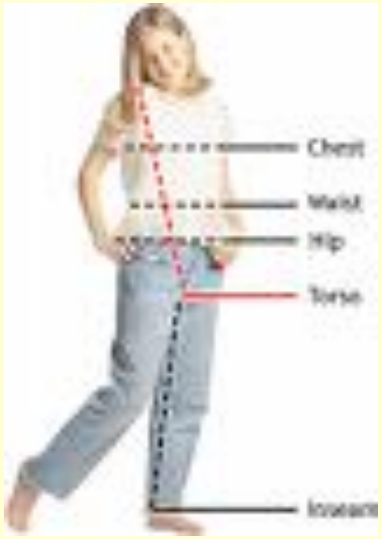
Teachers make the math rules

- **Meaning of letters:**

2b + 3 = 9

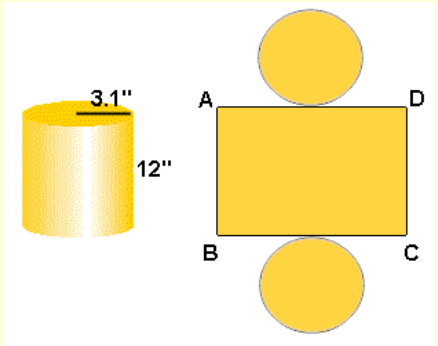
Math problems have only one correct answer

- **Girls' heights**



There is a math formula for everything

- **Length, Area and Volume of a Can**



Decimal points are not important as long the numbers are correct

- **Ratio problem:**

If a company can make 500 cartons in .8 hours, how many cartons can it make in 2 hours?

Plus and minus signs are not important as long the numbers are correct

Which information is more important to a sports fan, the score for the game (the numbers) or who is winning (the sign)?

#4

Teachers have misconceptions about what students know

- **Rounding Error:**

Round 65.23

- **Thalla and the house numbers**

#5

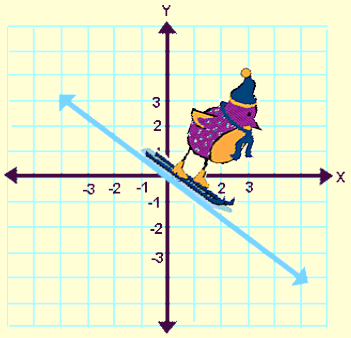
How to discover misconceptions

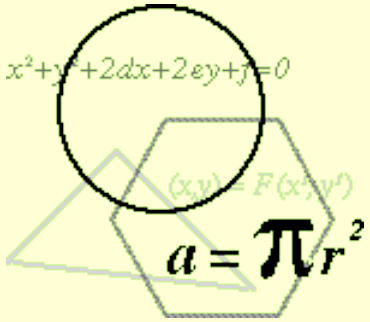
Look for misconceptions. It is easier to find something if you are looking for it.

- Emotional safety for students to ask and answer questions
- Reward critical thinking

Multiple Representations

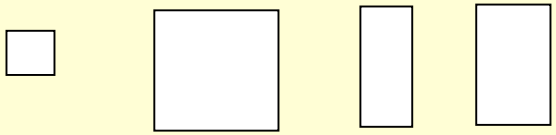
1. **Words**
2. **Table/Numbers**
3. **Graph**
4. **Symbols/Equations**
5. **Model**





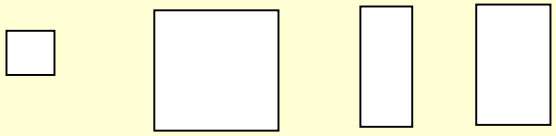
Lab experiment observations

- Cut out one square inch



Lab experiment observations

- Cut out one square inch



	Square Centimeter	Square Inch	1 “ long rectangle
Prealgebra	20%	40%	40%
Introductory Algebra	43%	43%	14%

Lab experiment observations

- **Measure from toe to toe**

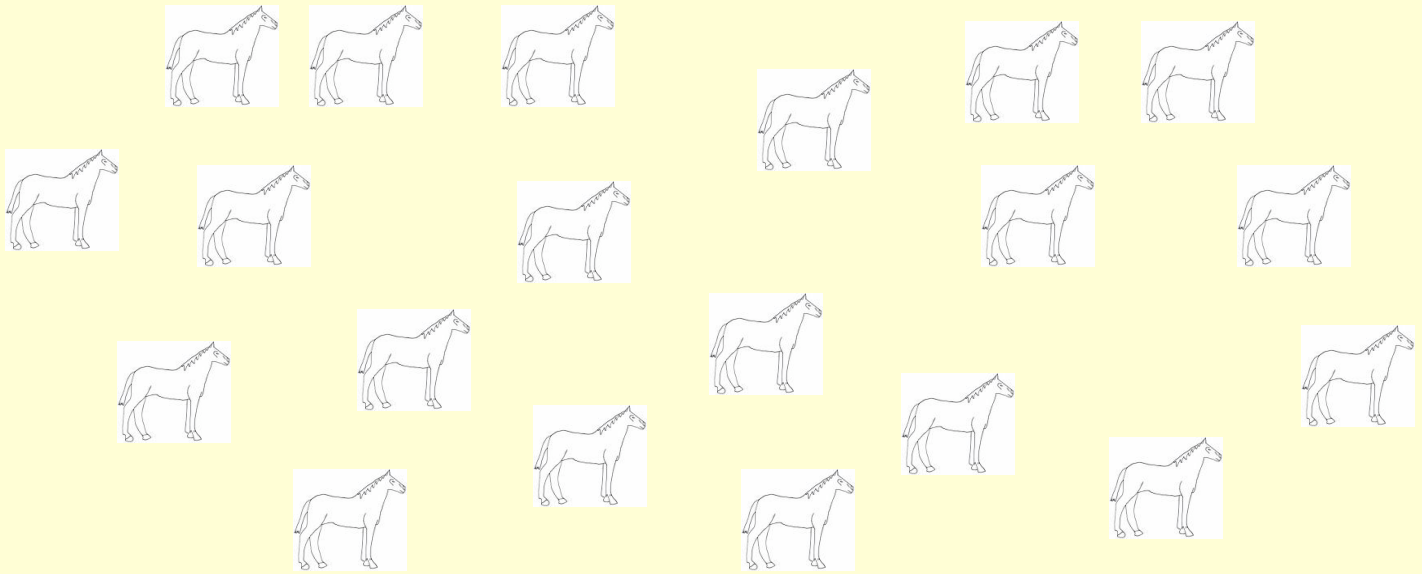
Lab experiment observations

- **Place decimals on a number line**

.6, .069, .609, .60

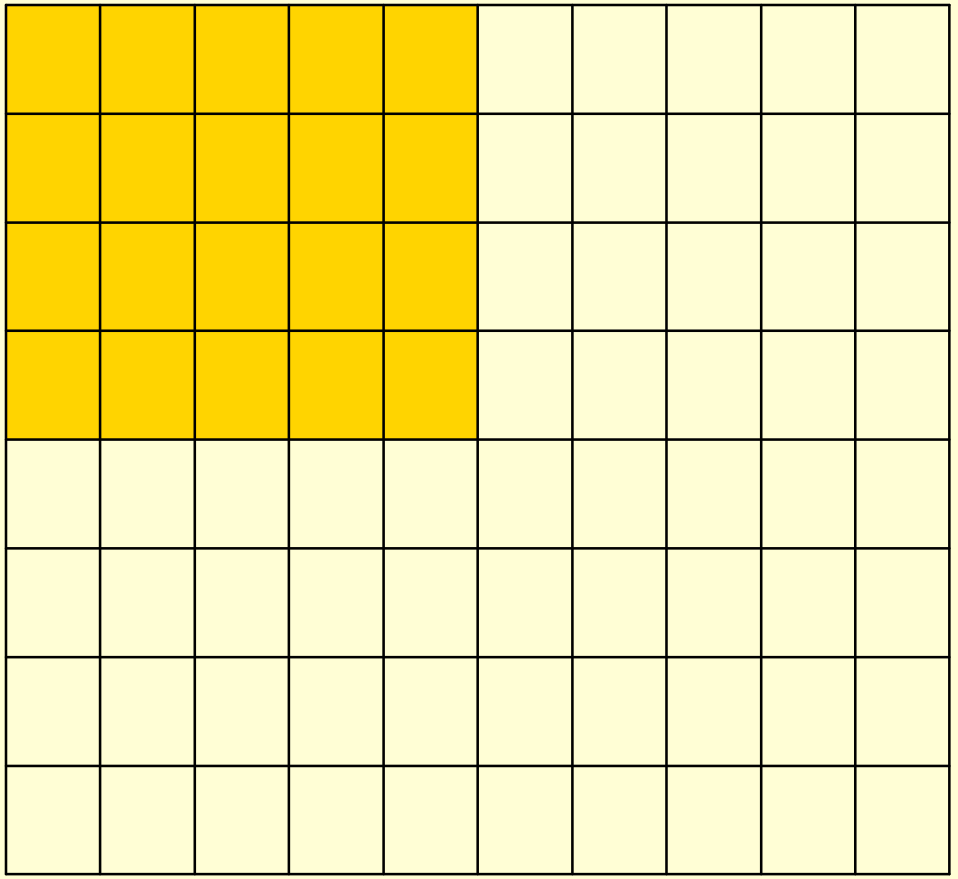
Lab experiment observations

- **Shade in 20% of these objects**



Lab experiment observations

- Represent the colored squares as a fraction, decimal and percent



Lab experiment observations

- Add and subtract fractions with spaghetti and a ruler

Ask questions and listen to student answers

Not “Does everyone understand?

Or “What is the Answer?”

Ask questions and listen to student answers

Instead

"How did you figure that out?"

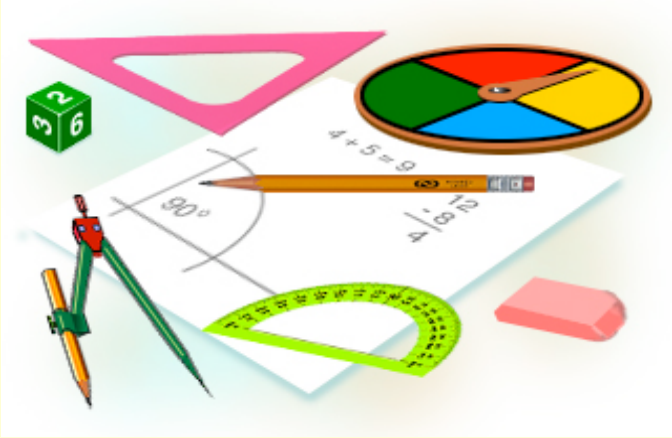
"How do you know your answer is correct?"

"Can you defend your reasoning?"

"What is the difference between v^2 and $2v$ if $v = 8$?

Ask questions and listen to student answers

"Because teacher gave me a rule" is not an acceptable answer.



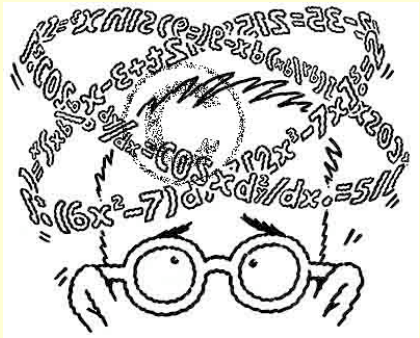
Ask questions and listen to student answers

Ask for more than one way to do a problem.

“Who found another way to do it?”

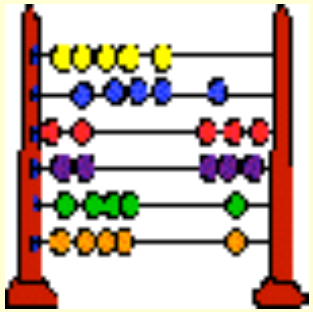
Ask questions and listen to student answers

When a student makes a mistake ask
"What makes you think that is true?"



Ask questions and listen to student answers

Ask “What have you always wanted to know about math but were afraid to ask?”



#6

How to correct misconceptions

Doing real stuff is not enough

- *Area as Length Plus Width:*

$$A = L + W$$

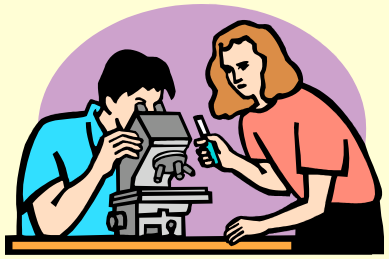
Use mental models

- Share models with other teachers. Each may have a different mental picture.
- *Mathpack: Models for signed numbers*

War, distance, canceling particles, up and down stairs, walking back and forth, piles and holes, elevator, water tank, weights and balloons, bank balance, football yardage, temperature

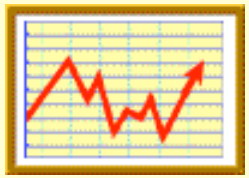
Develop number sense

- Understanding in context of personal experiences, connections, relevance to life
- Belief that math can be about something real



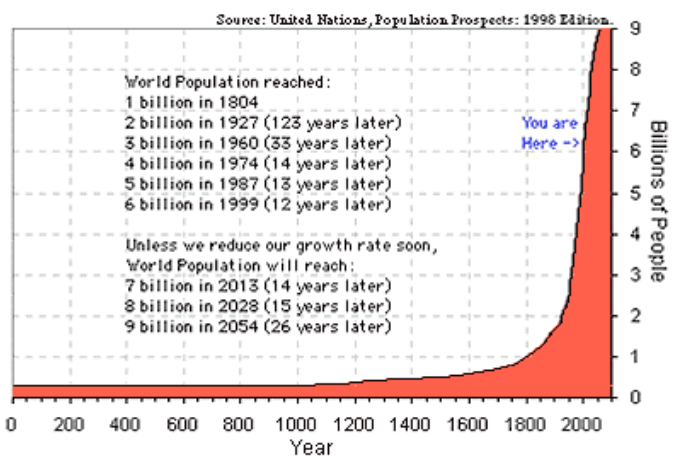
Personalize Learning

Provide memorable “aha” experiences



Encourage Ownership

Challenge students to create their own models



Repeat exposures to new terms

- Acquiring vocabulary needs 7 times in context
- Repeat definitions parenthetically for new vocabulary words: e.g. “Circumference (the distance around the circle)”

Repeat exposures to new terms

Tell students to “Say it when you do it”

- ***Metric Lab***

Keep a Diary

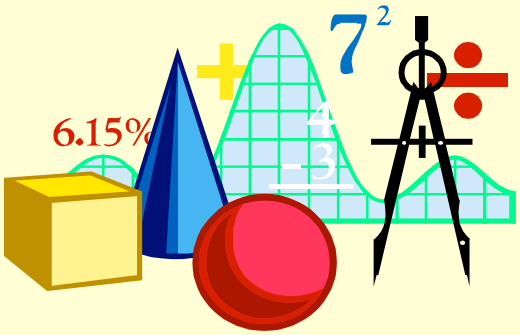
Keep a log of your students' misconceptions and how you corrected them

Only memorize with Understanding

Instead of a rule “move the decimal point two places to the left” memorize “the percent sign means per hundred or divide by 100”

www.cardells.net

A place to find math activities and resources



www.mathmisconceptions.cardells.net

A place to share stories and ideas

Keep on Digging