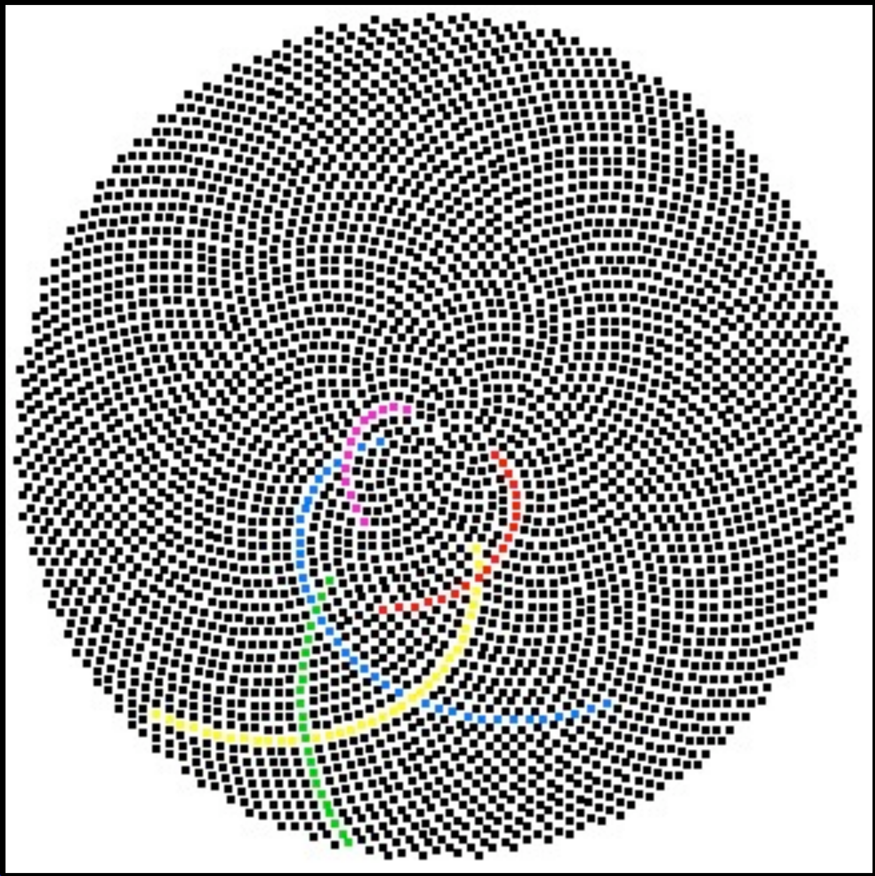
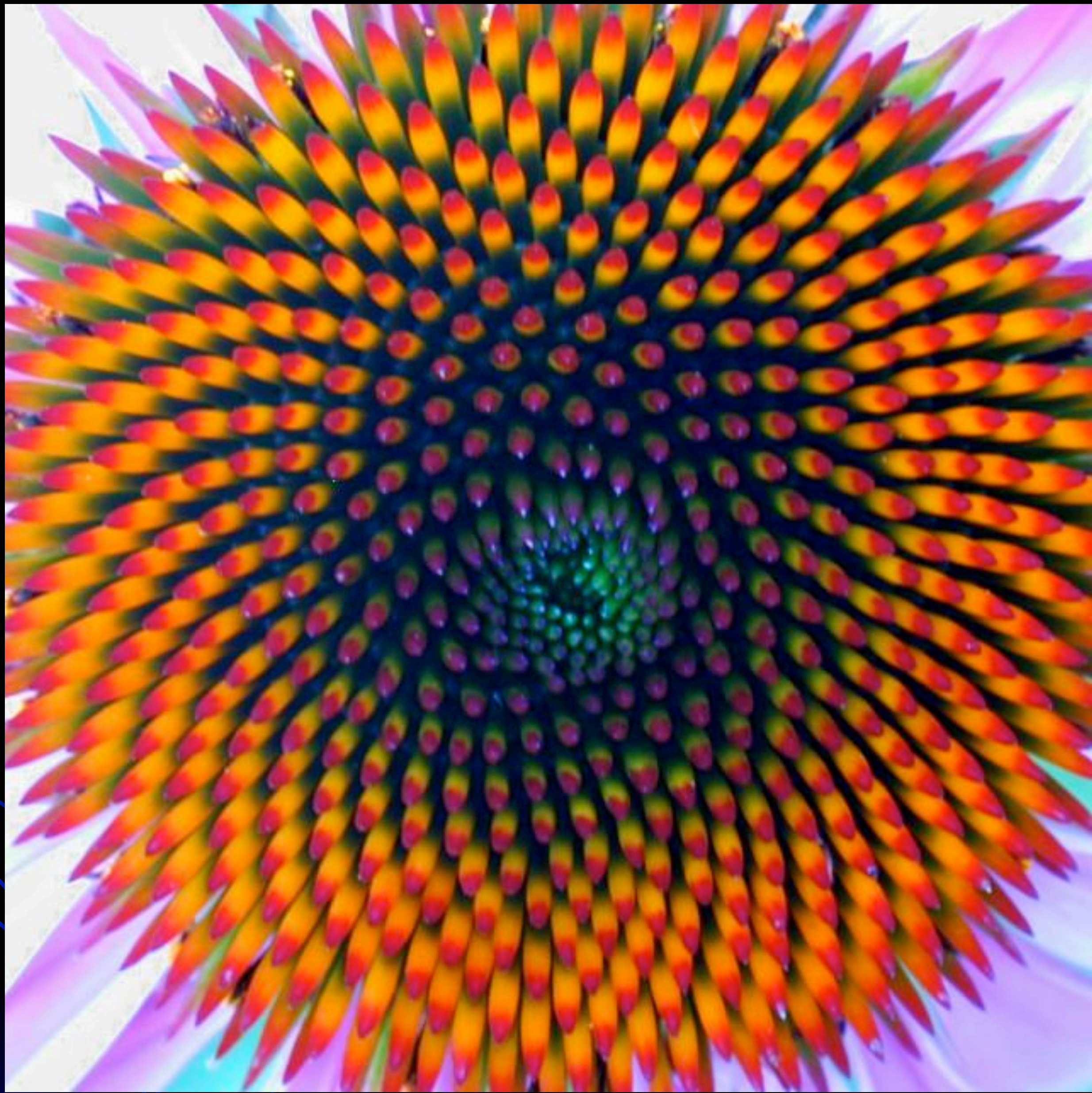
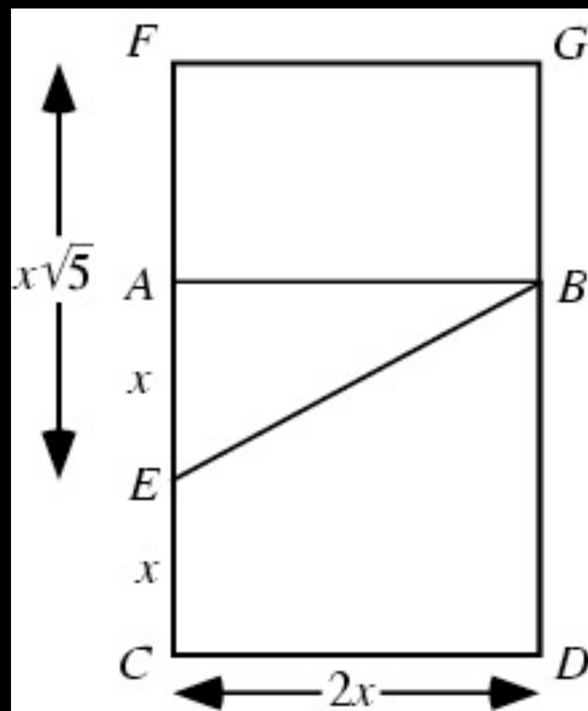
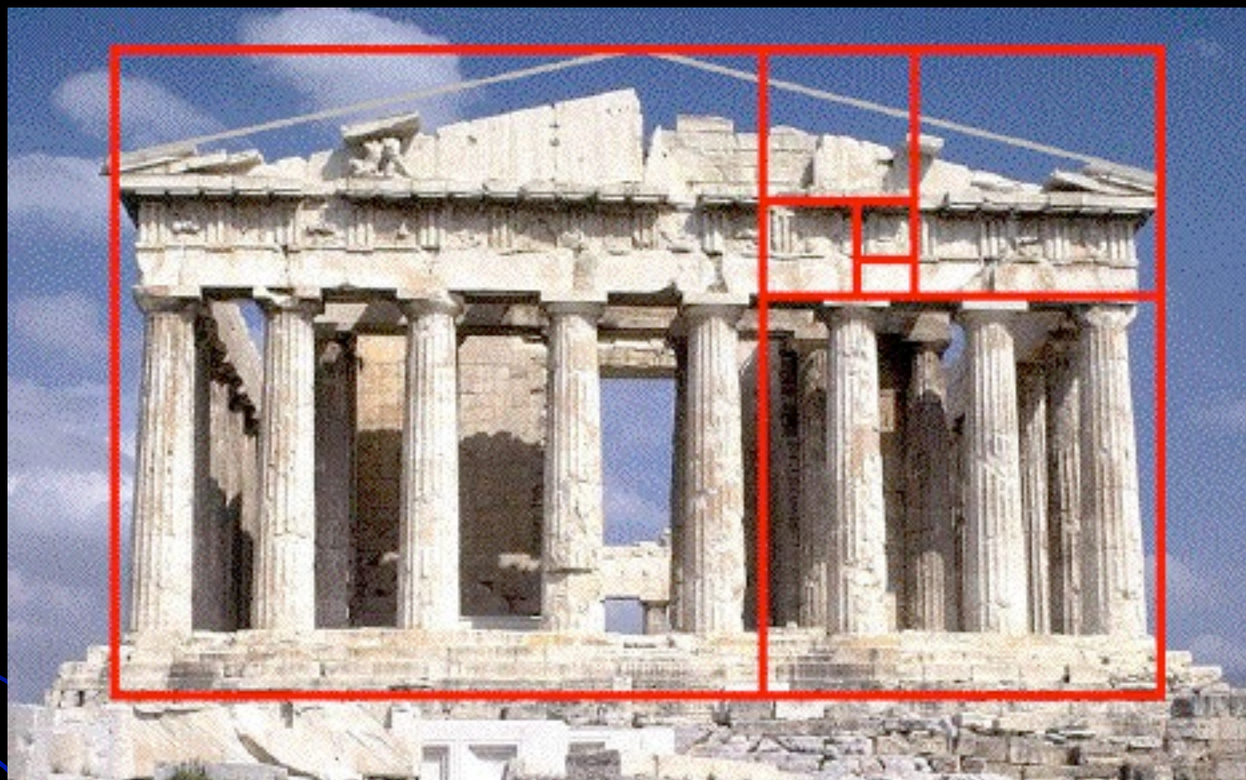
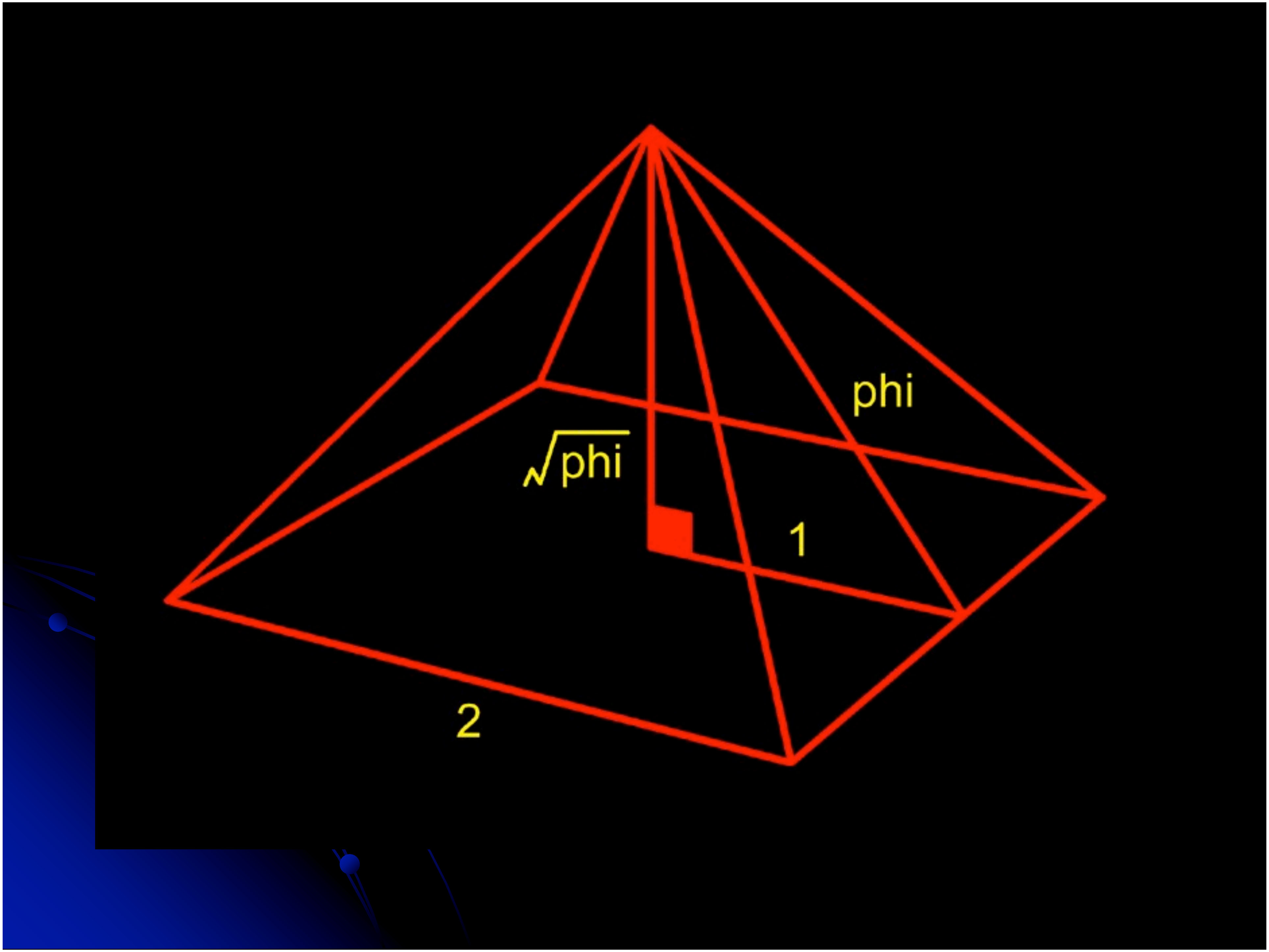
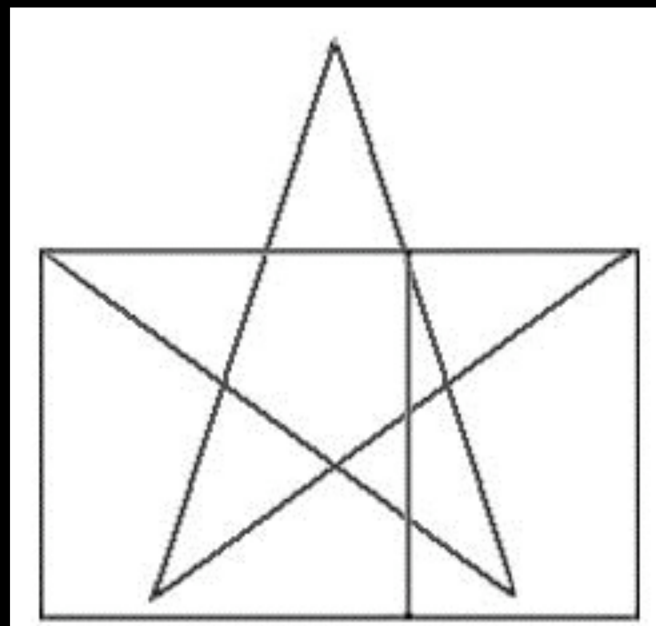


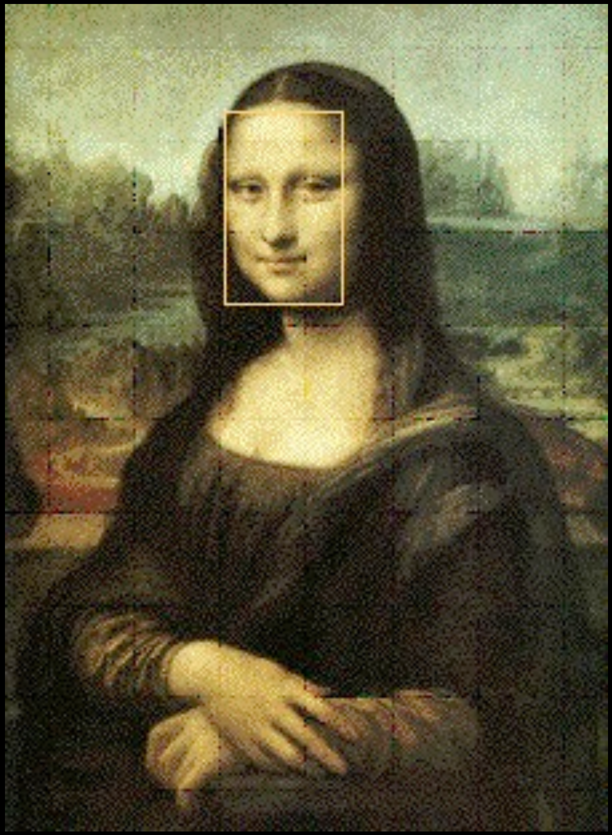


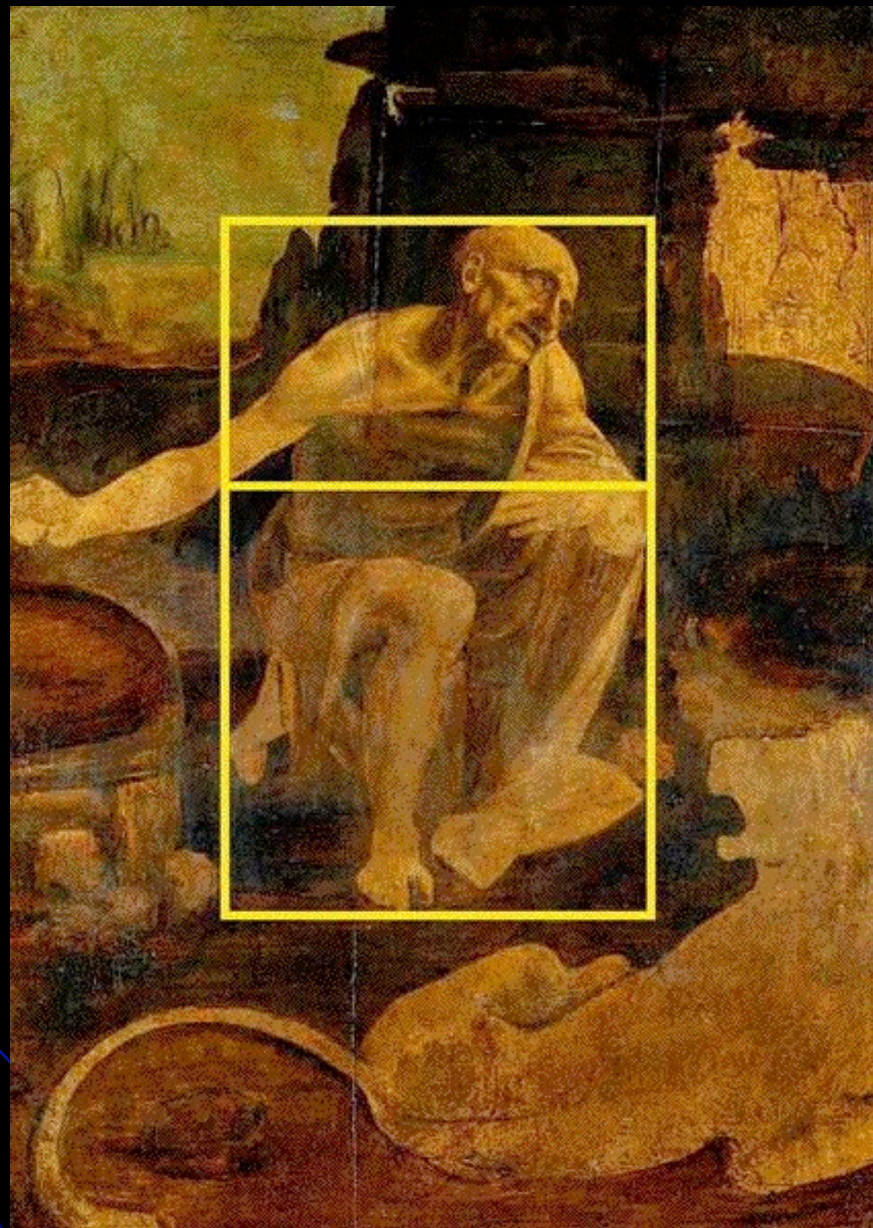

F
G
$x\sqrt{5}$
A
B
x
E
x
C
D
2x

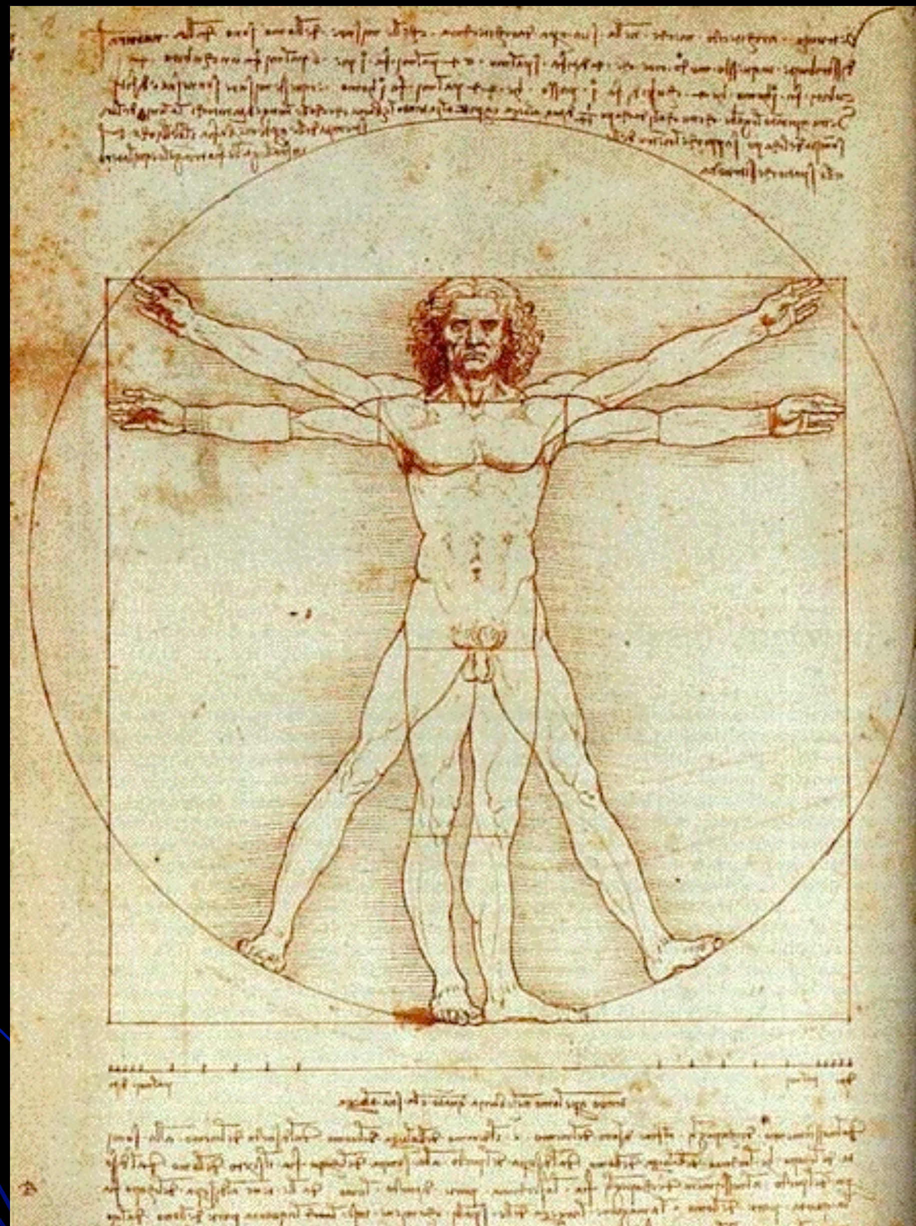


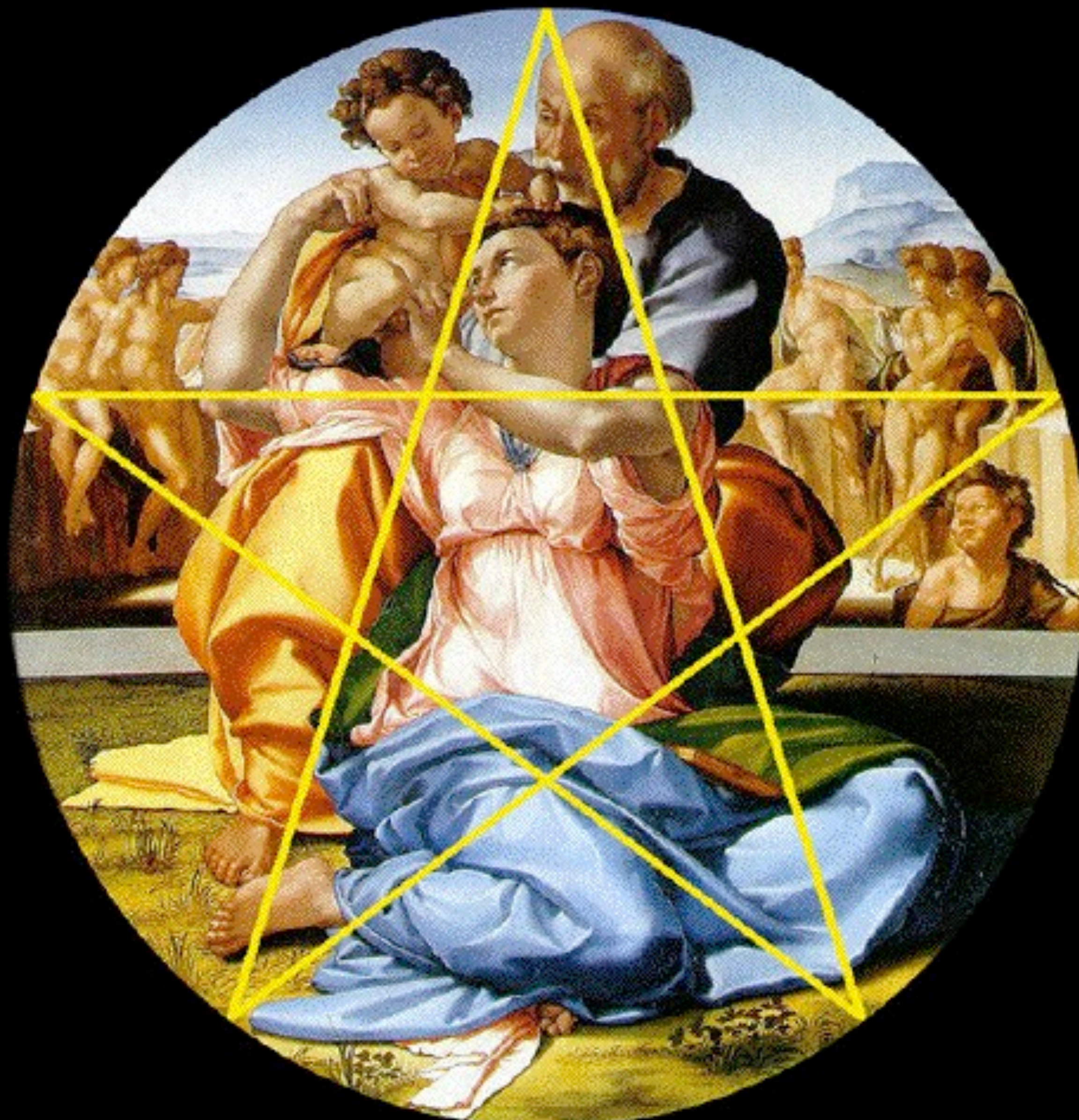

phi
√phi
1
2

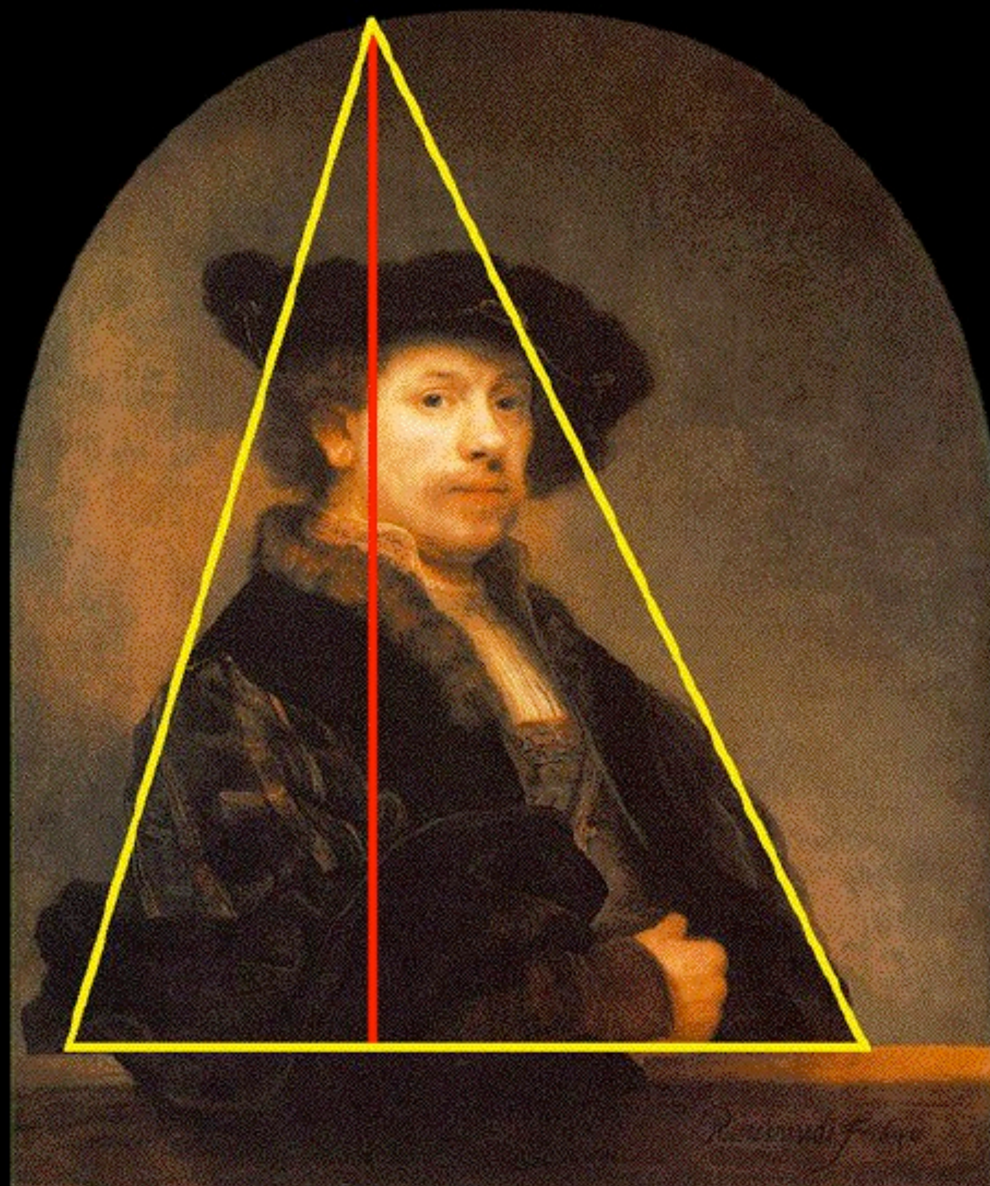












**Phi, $\Phi$, in arithmetic, coincidentally 1**

## $\Phi - 1 = 1 / \Phi$

Interestingly, phi minus one equals one divided by phi, ($\approx .6180339887$). This equivalence enables convenient computing and labeling of the side-lengths of the squares formed in a repeated partition of a golden rectangle.

These lengths can be calculated by subtracting the short side of the prior rectangle from its long side. Doing so confirms that consecutive squares are in the golden ratio, thus also are the resultant rectangles. Consider the sequence:

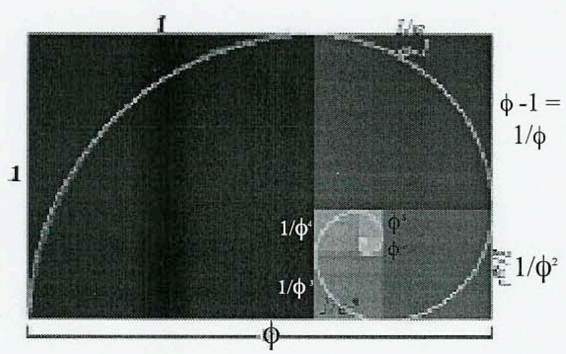
Side length of **first** square cut out, its short side: 1 unit

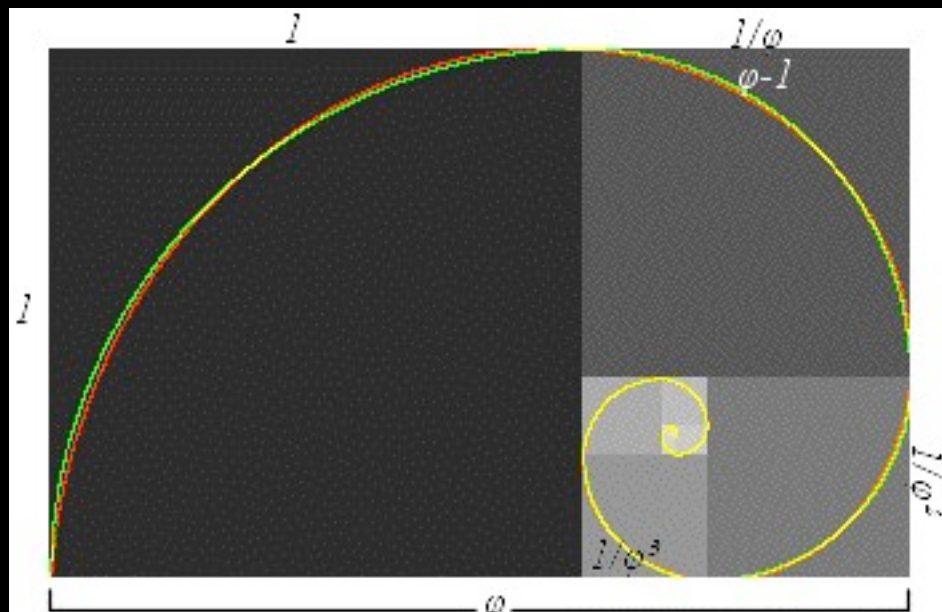
Side length of **second** square cut out, long side minus short: $(\Phi - 1)$, which is $(1 / \Phi)$ units.

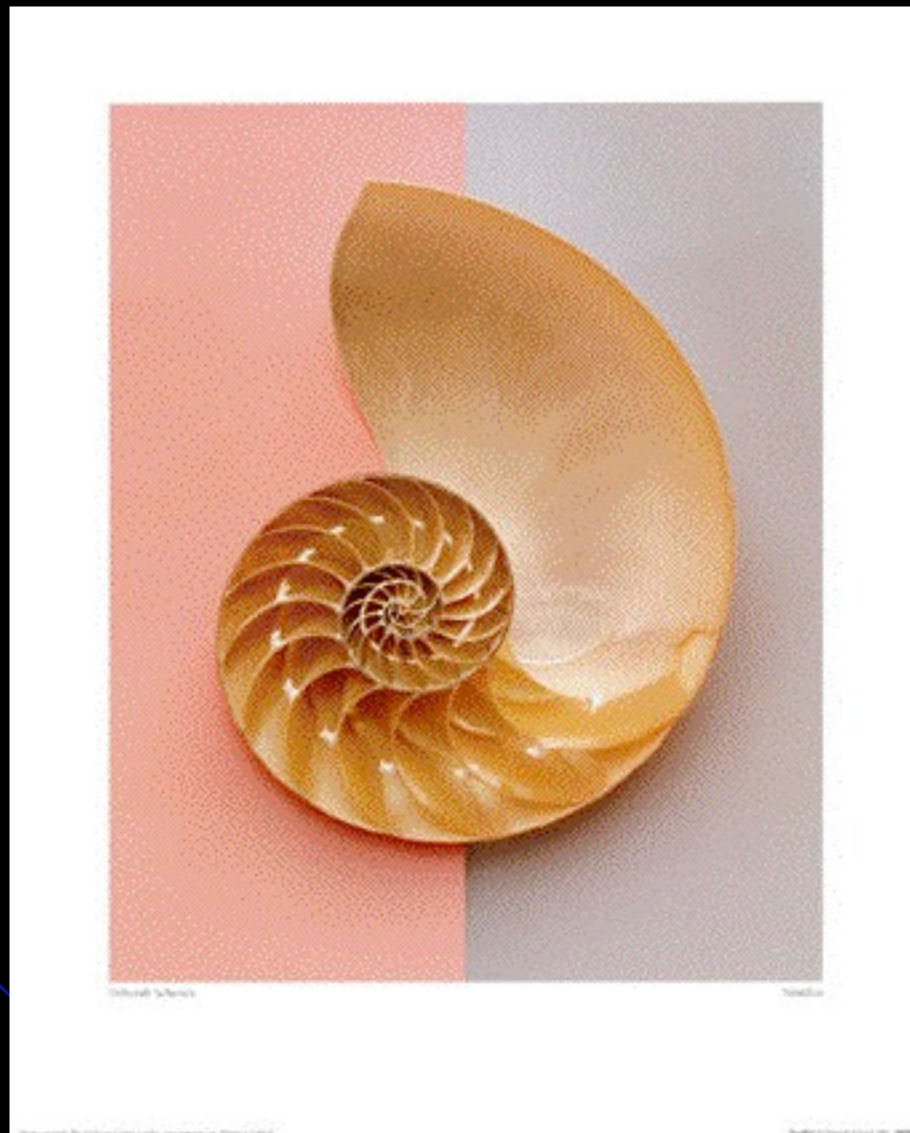
Side of **third** square cut out, long side minus short: $1 - (\Phi - 1) = 1 - (1 / \Phi) = [(\Phi - 1) / \Phi] = [(1 / \Phi) / \Phi] = \mathbf{1 / \Phi^2}$.

Side of **fourth** square cut out: $(\Phi - 1) - [1 - (\Phi - 1)] = 1 / \Phi - 1 / \Phi^2 = \mathbf{1 / \Phi^3}$.

**In general, the $n^{th}$ square will have side length $(1 / \Phi^{n-1}) = \Phi^{1-n}$.**




1
1/φ
φ-1
1
1/φ²
1/φ³
φ



Phi, Φ, in arithmetic, coincidentally 2

$$\Phi + 1 = \Phi^2$$

Also interestingly, phi plus one equals phi squared (≈ 2.6180339887). This equivalence enables us to express the powers of phi in linear terms, and brings us back to the fibonacci sequence (1 1 2 3 5 8 13 21 34 ...). Consider the sequence:

$$\Phi^2 = \Phi + 1$$

$$\Phi^3 = \Phi^2 * \Phi = (\Phi. + 1)\Phi = \Phi^2 + \Phi = (\Phi + 1) + \Phi = \mathbf{2\Phi + 1}$$

$$\Phi^4 = \Phi^3 * \Phi = (2\Phi. + 1)\Phi = 2\Phi^2 + \Phi = 2(\Phi + 1) + \Phi = \mathbf{3\Phi + 2}$$

$$\Phi^5 = \Phi^4 * \Phi = (3\Phi. + 2)\Phi = 3\Phi^2 + 2\Phi = 3(\Phi + 1) + 2\Phi = \mathbf{5\Phi + 3}$$

$$\Phi^6 = \Phi^5 * \Phi = (5\Phi. + 3)\Phi = 5\Phi^2 + 3\Phi = 5(\Phi + 1) + 3\Phi = \mathbf{8\Phi + 5}$$

If we label the terms of the fibonacci sequence by $F_n$,
then we see that the powers of phi can be expressed linearly as above,

**and in general:** $\Phi^n = F_n \bullet \Phi + F_{n-1}$